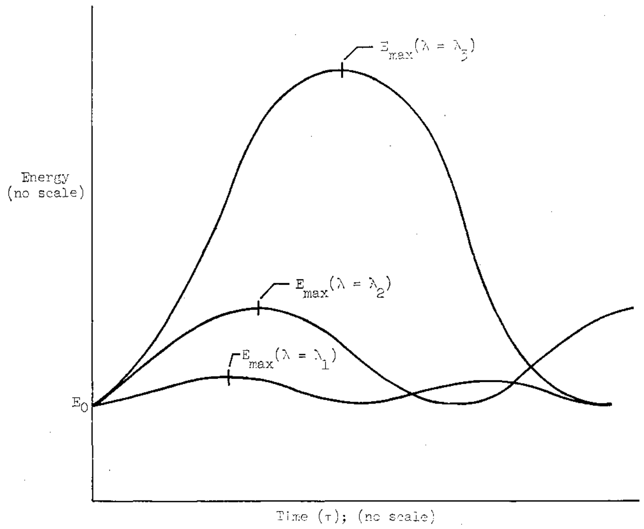

Fig. 2 Membrane system energy vs time. ($\lambda_3 > \lambda_2 > \lambda_1$).

the authors of the energy response of a plate subjected to supersonic gas flow had shown that the total plate energy response can be used to indicate stability.

A typical plot of the total membrane energy vs time, for the nonlinear case considering no damping, is shown in Fig. 2. The initial energy E_0, which is associated with the statically deflected membrane of shape w_0, is seen to be magnified by the action of the airflow to a maximum value E_{max}. Increasing the value of dynamic pressure parameter λ as represented by λ_1, λ_2, and λ_3 was found to give a similar energy response, for given initial conditions, except that the period of oscillation and the maximum value of the energy were changed. The change of the maximum energy with the dynamic pressure parameter λ is shown in Fig. 3. The maximum membrane energy was found to vary approximately exponentially with λ, but no instability was found.

The finite-difference equations associated with the linear case ($T = T_0$) were also solved by the same numerical integration procedure used in the nonlinear case. For the linear case a critical value of the dynamic pressure parameter was found to be associated with each number of grid points n. At this critical value of λ the system energy associated with the linear solution increased without bound; for the same value of λ, the corresponding solution for the nonlinear problem showed no departure from the energy response characteristics presented in Fig. 2, indicating that the nonlinearities are stabilizing. The critical value of λ_n associated with a particular number of grid points increases with n

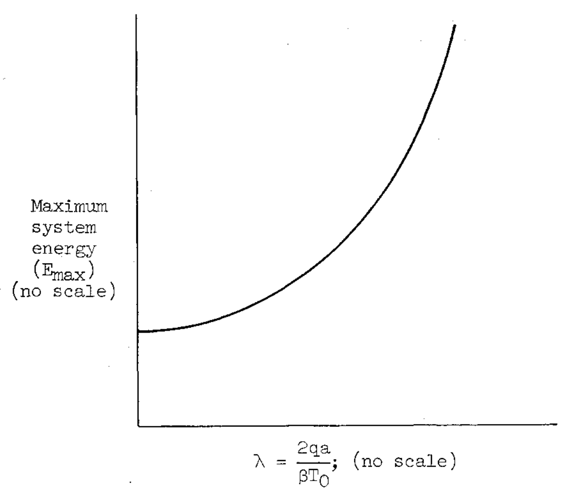

Fig. 3 Variation of maximum system energy with dynamic pressure parameter, λ.

and was found to approach infinity as n approaches infinity. It is of interest that a Galerkin solution to the linearized membrane problem also gives a critical value of λ for a finite number of modes. As was pointed out in Ref. 2, the critical value of λ approaches infinity as the number of modes approaches infinity.

Concluding Remarks

In this study, the response of a membrane in supersonic flow has been considered as an initial value problem. The results show that the effect of the nonlinearity (increasing tension with deflection) is stabilizing from the standpoint of classical stability for all finite values of the dynamic pressure for the case of a finite number of grid points. Both the period and magnitude of the response were reduced compared to the linear solution for a given value of the dynamic pressure parameter λ. However, from observation a membrane can appear to be fluttering, since the magnitude of an initial disturbance is magnified by an exponential function of λ and x. The experimental determination of any flutter behavior of membrane-like panels may thus be difficult, since flow disturbances can cause a response that resembles the classical flutter behavior.

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Flow Field for Sonic Jet Exhausting Counter to a Hypersonic Mainstream

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Nomenclature

A	=	area
C_D	=	drag coefficient
D_j	=	distance of jet shock from nozzle exit
M	=	Mach number
p	=	pressure
q	=	dynamic pressure
R	=	radius
T	=	thrust
U	=	velocity
ρ	=	density
γ	=	ratio of specific heats

Subscripts and Superscripts

1	=	jet throat
2	=	downstream jet station (see Fig. 1)
∞	=	freestream conditions
i	=	interface or contact surface between two flows
j	=	jet conditions
t	=	stagnation conditions
$(\ )'$	=	conditions after a normal shock

INFORMATION reported in Ref. 1 has provided an explanation for two different types of mainstream shock displacements for jets exhausting counter to a mainstream flow.

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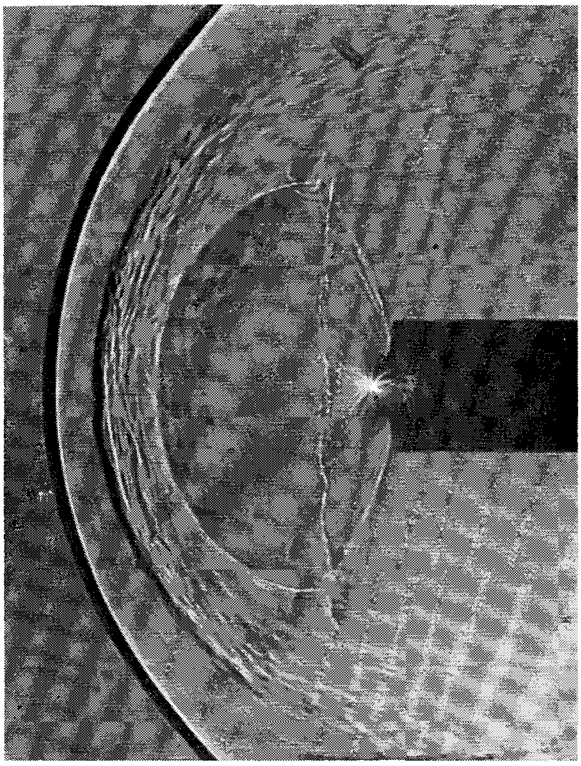
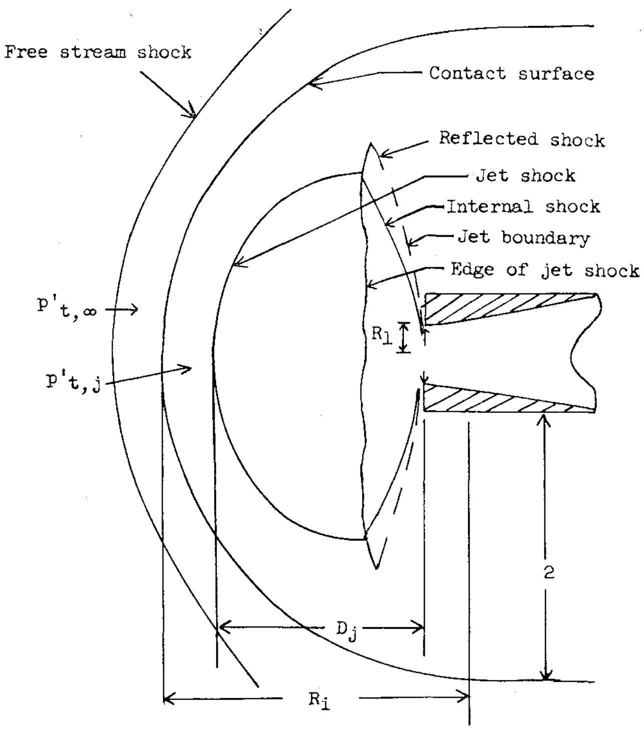
Data for one of these types at a freestream Mach number of 2.5 and 2.9 have been obtained in Refs. 2 and 3 where it was shown that a sonic jet would overexpand and go through a normal shock in order to satisfy the centerline pressure conditions. Additional data obtained from photographs for this type of flow for a sonic jet exhausting counter to Mach number 6.0 and 8.5 mainstream flow are presented in this note. An extension of the theory for this flow field is also presented.



a) Shadowgraph



b) Schematic flow

Fig. 1 Flow field for a sonic jet, $M_\infty = 6.0$, $p_{t,j}/p'_{t,\infty}$ 33.3.

This study was conducted in the 20-in. Hypersonic Tunnel Section at Langley Research Center.

Figure 1a shows a typical shadowgraph taken at a freestream Mach number of 6.0. Figure 1b is a sketch that describes the photograph and defines some of the symbols used in this note. The jet expands from sonic flow in the nozzle to a high Mach number, experiences a normal shock on the axis and decelerates subsonically to a mutual stagnation point. The main flow also encounters a normal shock on the axis; and the recovered stagnation pressure for both flows, $p'_{t,\infty}$ and $p'_{t,j}$ for the mainstream and jet, respectively, must be equal regardless of the jet pressure. The Mach number (M_j) of the jet flow just before the shock occurs must therefore be a value that will satisfy this pressure as the jet total pressure ($p_{t,j}$) is varied. The value of M_j on the centerline can be found from normal shock equations; for example, for perfect gas flow with $\gamma = 1.4$, the following equation must be satisfied:

$$\left(\frac{6M_j^2}{M_j^2 + 5}\right)^{7/2}\left(\frac{6}{7\,M_j^2 - 1}\right)^{5/2} = \frac{p'_{t,j}}{p_{t,j}} = \frac{p'_{t,\infty}}{p_{t,\infty}}\frac{p_{t,\infty}}{p_{t,j}} \quad (1)$$

Previous data and new data (Fig. 2) illustrate that the jet position on the centerline at which the jet flow experiences a normal shock can be correlated as a function of $p_{t,j}/p'_{t,j}$ (or $p_{t,j}/p'_{t,\infty}$) for all Mach numbers tested. The experimentally measured jet shock position D_j was taken from the photographs and is defined in Fig. 1b. The results of Fig. 2 would be expected since theoretical considerations[4, 5] have shown that the Mach number distribution on the centerline of a sonic jet remains constant up to the exit normal shock (or before the compressions from the jet boundary intersect the centerline) regardless of the jet pressure ratio. Any increase in the jet pressure ratio merely extends the centerline Mach number to higher values without changing the already established centerline Mach number distributions before the compressions or shock. Figure 2 also shows that the values of D_j/R_1 can be predicted theoretically using the values of M_j obtained from Eq. (1) and the known centerline Mach number variation previously obtained for a sonic jet exhausting into free air (see Ref. 4 for centerline Mach number distribution).

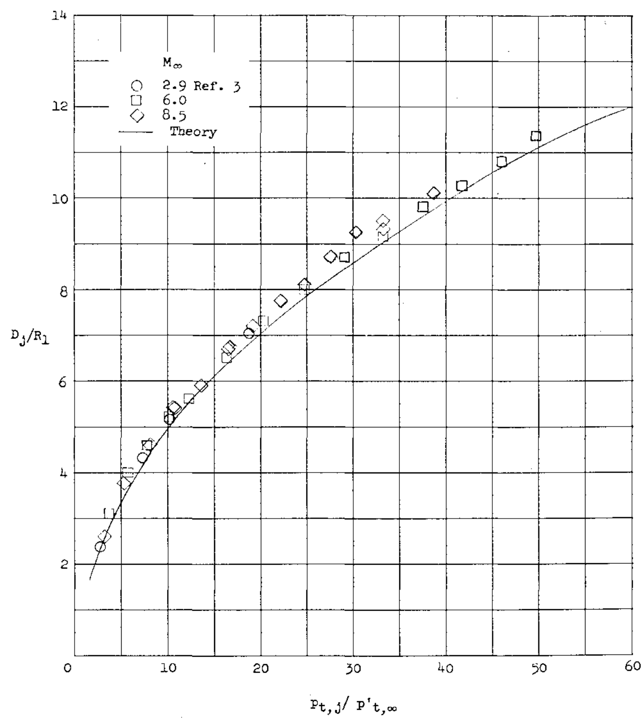


Fig. 2 Ratio of jet-shock radii for various jet pressures.

From the photographs at $M_\infty = 6.0$ and $M_\infty = 8.5$, it has been observed that the jet shock and a large part of the contact surface (see Fig. 1) can be very closely approximated by spherical segments. The static pressures that exist at the contact surface can, therefore, be estimated by assuming that the contact surface is a blunt hemispherical body and that the mainstream flow must expand around it. A simple approximate method has also been established for predicting the radius of the contact surface, which is based on equating the axial forces on the contact surface caused by the mainstream and jet flows. That is, the contact surface is treated as a hemispherical shell held in equilibrium by the two flows. The mainstream force or drag was found by assuming a Newtonian pressure distribution on the contact surface, which was taken to be a hemisphere. This force is then simply

$$D = C_D q_\infty (\pi R_i^2) \tag{2}$$

The thrust on the interface shell produced by the jet (again taken to be hemispherical) is equal to the thrust of the jet gas exhausting at some arbitrary downstream station [see Fig. 1b] plus the thrust of the jet as it exhausts at the jet throat. These values are

$$T_i = \rho_2 U_2^2 A_2 + (p_2 - p_\infty)A_2 + \rho_1 U_1^2 A_1 + (p_1 - p_2)A_1 \tag{3}$$

However, from conservation of mass

$$\rho_2 U_2 A_2 = \rho_1 U_1 A_1 \tag{4}$$

and, making the assumption that $p_2 = p_\infty$, which is known to be reasonable for the type of jet shown in Fig. 1, Eq. (3) can be rewritten as

$$T_i = (\rho_1 U_1 A_1)(U_2 + U_1) + (p_1 - p_\infty)A_1 \tag{5}$$

When $M_1 = 1.0$ this can be rewritten as

$$T_i = p_1 A_1[\gamma + 1 + \gamma(U_2/U_1) - (p_\infty/p_1)] \tag{6}$$

However, in all cases $(p_\infty/p_1) \ll 1$, therefore

$$T_i = \gamma p_1 A_1[(\gamma + 1/\gamma) + (U_2/U_1)] \tag{7}$$

Equating the thrust to the drag

$$C_D q_\infty(\pi R_i^2) = \gamma p_1[(\gamma + 1/\gamma) + (U_2/U_1)]\pi R_1^2 \tag{8}$$

or

$$\frac{R_i}{R_1} = \left[\gamma\left(\frac{\gamma+1}{\gamma} + \frac{U_2}{U_1}\right)\frac{p_1}{C_D\, q_\infty}\right]^{1/2} \tag{9}$$

since $p_1 = 0.528\, p_{t,j}$ for a sonic jet and $p'_{t,j} = p'_{t,\infty}$

$$\frac{R_i}{R_1} = \left[\frac{(0.528)}{C_D}\,\gamma\left(\frac{\gamma+1}{\gamma} + \frac{U_2}{U_1}\right)\left(\frac{p'_{t,\infty}}{q_\infty}\right)\left(\frac{p_{t,j}}{p'_{t,j}}\right)\right]^{1/2} \tag{10}$$

Everything on the right-hand side of the equation is practically constant for hypersonic flow except $p_{t,j}/p'_{t,j}$, so that $R_i/R_1 \propto (p_{t,j}/p'_{t,j})^{1/2}$. A plot of R_i/R_1 vs $p_{t,j}/p'_{t,j}$ is shown in Fig. 3 along with measured values taken from the photographs. The calculations for Fig. 3 were made assuming $\gamma = 1.4$, $C_D = 0.9$ (from modified Newtonian), and U_2/U_1 equal to both 2.45 and 1.8. The value of 2.45 is the maximum possible velocity ratio for a sonic jet and the value of 1.8 is an arbitrary lower limit. The lower limit is based on calculations of U_2 assuming an average $p'_{t,j}$ (for the complete jet shock) as the stagnation pressure for the flow at station 2 where the flow has expanded to freestream static conditions. It can be seen from Fig. 3 that Eq. (10) can be used to obtain a reasonable prediction of R_i/R_1 over a wide range of M_∞. The results of Eq. 10 do not definitely locate the contact surface since its center is unknown. (Experimentally it has been noted that the centers are always before the nozzle exit as indicated in Fig. 1b.)

In conclusion, therefore, it has been shown from the experimental data that the jet shock size and contact surface size are approximately spherical segments and can be found for any freestream Mach number when correlated as a function of $p_{t,j}/p'_{t,\infty}$. Further, theoretical methods have been established which predict the standoff distance of the jet shock and the radius of the contact surface. Thus, the flow field for the counter jet can largely be constructed analytically from known pressure conditions and freestream Mach number.

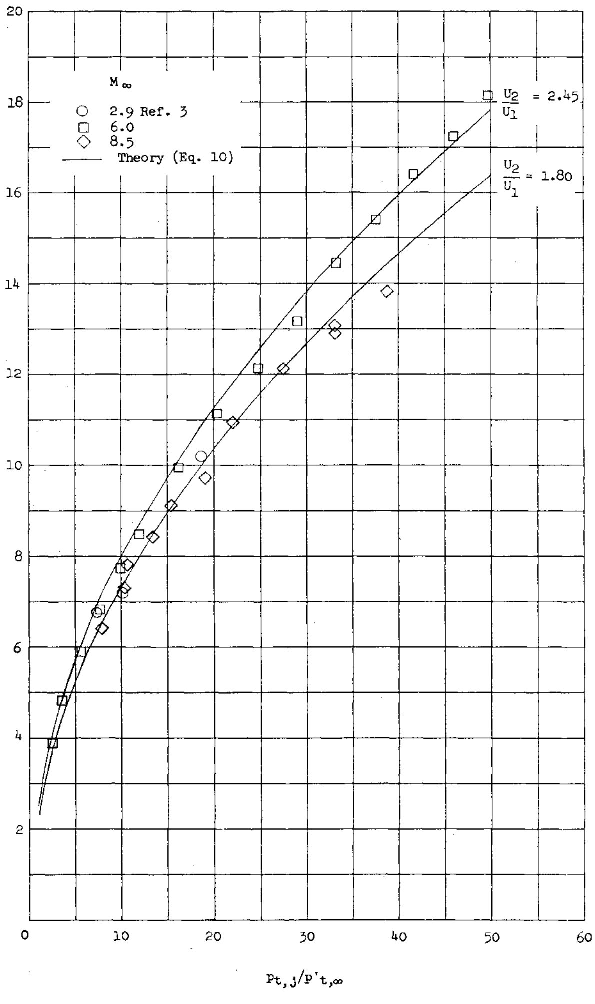


Fig. 3 Ratio of jet-contact radii for various jet pressures.

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